

# NOISE-PARAMETER UNCERTAINTIES FROM MONTE CARLO SIMULATIONS

J. Randa<sup>h</sup> and W. Wiatr<sup>l</sup>

<sup>h</sup>Radio Frequency Technology Division  
National Institute of Standards and Technology  
Boulder, CO, U.S.A

<sup>l</sup>Warsaw University of Technology  
Institute of Electronic Systems  
Warsaw, Poland

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## I. INTRODUCTION



### OVERVIEW

- Have written a simulator for noise-parameter measurements. Intended for:
  - test fitting program
  - experiment with different measurement strategies, computing uncertainties with simulated data. E.g, can vary number of terminations, pattern of ' 's, inclusion of reverse configuration, ...
  - evaluate uncertainties in actual measurements



- Point of this paper is to use Monte Carlo simulations to compute the uncertainties in the noise parameters for known uncertainties in the underlying quantities,  $T_{\text{source}}$ ,  $\Gamma_{\text{source}}$ ,  $P_{\text{meas}}$ ,  $T_{\text{amb}}$ , S-parameters, connector variability.



- Related work:
  - Adamian, unpublished lecture notes, 1991
  - Simulations to study different patterns for  $\Gamma$ 's
    - Davidson, Leake, & Strid, IEEE Trans. MTT, vol. 37, no. 12, pp. 1973 – 1978, 12/89
    - Schmatz, Benedickter, & B@chtold, *Digest of the 51<sup>st</sup> ARFTG Conference*, Baltimore, MD, 6/98.
    - Van den Bosch & Martens, IEEE Trans. MTT, vol. 46, no. 11, pp. 1673 – 1678, 11/98
  - Agilent web-based uncertainty calculator for noise figure (not noise parameters)
  - Presumably others of which I'm unaware

## OUTLINE

- Introduction
  - Overview
  - Outline
- Calculation
  - Process to be simulated
  - The simulator
- Results
- Discussion

## II. CALCULATION

### PROCESS TO BE SIMULATED

- Noise figure of amplifier,  $F = \frac{\text{Noise out}}{\text{Noise in}} \Big|_{\text{Noise in} = kT_0}$

$$N_{in} = k_B T_0 \rightarrow \boxed{G} \rightarrow N_{out} = G k_B T_0 + N_{amp}$$

- Equivalent input temperature  $T_e$ ,  $N_{amp}/Gk_B T_e$

$$\begin{matrix} T_e \\ T_0 \end{matrix} \rightarrow \boxed{G} \rightarrow T_{out} = G(T_0 + T_e)$$

- Then  $F = (T_0 + T_e) / T_0$



- Complication:  $F = F('_{\text{source}})$ ,  $T_e = T_e('_{\text{source}})$
- So parameterize dependence on '<sub>source</sub>:

– IEEE

$$T_e = T_{e,\min} \cdot t \frac{*G_G ! G_{opt}^{*2}}{(1 ! *G_G^{*2})^2 * 1 \% G_{opt}^{*2}}$$

$$t = 4 \frac{R_n}{Z_0} \quad G = '_{\text{source}}$$

- Noise matrix in wave representation

$$T_e = \frac{*G_G^{*2}}{(1 ! *G_G^{*2})} X_1 \% \frac{*1 ! G_G S_{11}^{*2}}{(1 ! *G_G^{*2})} X_2 \% \frac{2}{(1 ! *G_G^{*2})} Re[(1 ! G_G S_{11})^C G_G X_{12}] .$$

where  $X_1 / <*\hat{b}_1^{*2}>$ ,  $X_2 / <*\hat{b}_2/S_{21}^{*2}>$ ,  $X_{12} / <\hat{b}_1(\hat{b}_2/S_{21})^C>$



- Can relate one set to the other.
- Standard measurement method is to
  - write equation for output power (or noise figure, or noise temperature) in terms of noise parameters and other variables ( $'_{\text{source}}$ ,  $T_{\text{source}}$ , S-parameters)
  - measure output for many different sources (different '<sub>source</sub> &  $T_{\text{source}}$ )
  - fit for noise parameters
- Monte Carlo simulation provides good way to evaluate uncertainties in noise parameters due to uncertainties in other variables.

## THE SIMULATOR

- General idea of M-C simulation:
  - assume “true” values of noise parameters and underlying measured quantities ( $\epsilon_{\text{source},i}$ ,  $T_{\text{source},i}$ , S-parameters, measured powers).
  - Choose a random “measured” value for each measured quantity (Gaussian distribution, centered at true value,  $F = \text{uncert in that meas.}$ )

- Compute true value of  $P_{\text{out},i}$  for each source  $i$ ; generate measured value (allow connector variation)

$$P_{\text{out}} = (k_B B) |S_{21}|^2 \left\{ \frac{(1 - |\Gamma_G|^2)}{|1 - \Gamma_G S_{11}|^2} T_G + \left| \frac{\Gamma_G}{1 - \Gamma_G S_{11}} \right|^2 X_1 + X_2 + 2 \operatorname{Re} \left[ \frac{\Gamma_G}{1 - \Gamma_G S_{11}} X_{12} \right] \right\}$$

- Analyze “measured”  $P_{\text{out},i}$  to get  $T_{\min}$ ,  $t$ ,  $\epsilon_{\text{opt}}$ ,  $G$  (used W.W.’s fitting routine)
- Repeat many times (100 - 1000).
- Standard uncert in  $T_{\min}$  is standard dev. of sample of “measured”  $T_{\min}$ ’s, etc.

- Particular case analyzed:

NIST  
NOISE

$|S_{21}|^2 \equiv G_0 = 2399$  (33.80 dB)  
 $T_{e,min} = 109.6\text{ K}$  ( $F_{min} = 1.392$  dB)  
 $G_{opt} = 0.050 + 0.142j$   
 $t = 176.3\text{ K}$   
 $S_{11} = 0.0181 - 0.1215j$   
 $S_{22} = 0.1371 - 0.0300j$   
 $S_{12} = 0.0018 + 0.0007j$   
 $T_{G,1} = 9920\text{ K}$ ,  $\gamma_{G,1} = 0.028070 + 0.022718j$   
 $T_{G,i} = 296\text{ K}$  ( $i = 2 - 13$ )

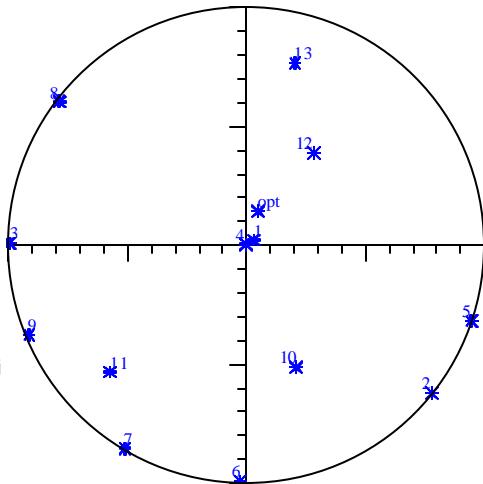
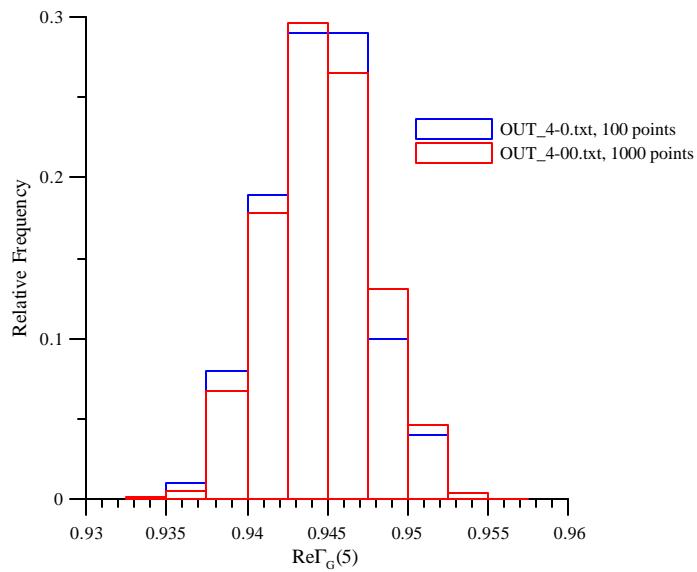
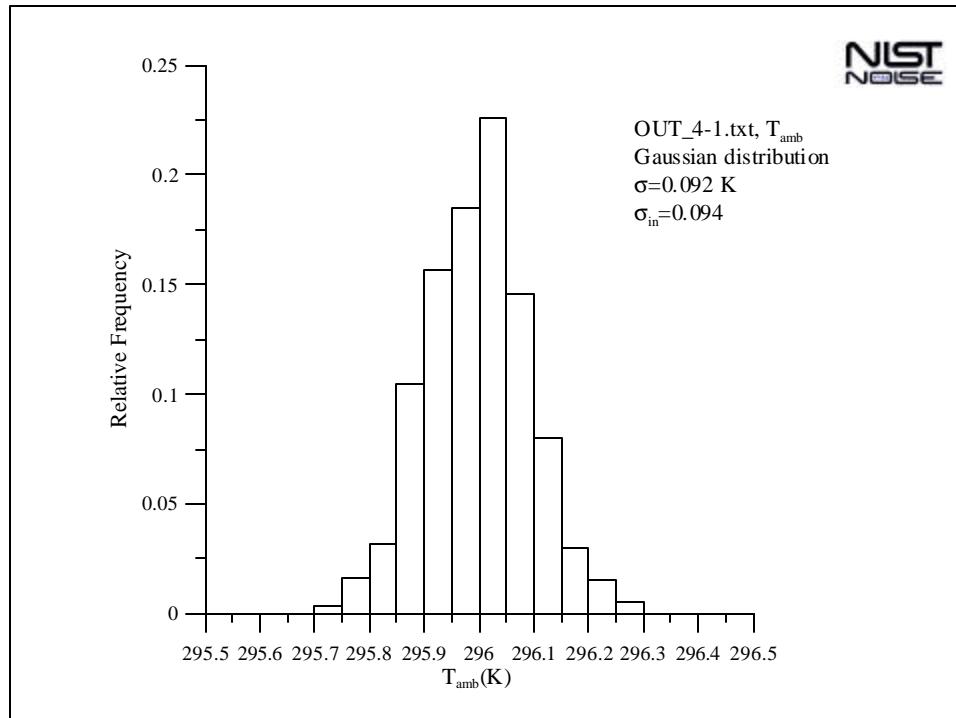


Fig. 1. Distribution of reflection coefficients of terminations in and on unit circle.

- Representative distributions:

NIST  
NOISE





### III. RESULTS

- Baseline set of uncertainties, used for comparison:

$$\begin{aligned}
 F_{S21} &= 0.01 \\
 F_{\cdot} &= 0.002 (' \#0.5), 0.003 (' >0.5) \\
 F_{\text{con}} &= 0.001 \\
 F_{\text{Ta,frac}} &= 0.005 \\
 F_{\text{Th,frac}} &= 0.005 \\
 F_{\text{n frac}} &= 0.001
 \end{aligned}$$

- Small S-parameters treated same as  $'$ 's.
- Actually, base set corresponds to very good uncertainties.



– Results for baseline set:

$$G_0 = 2400 \pm 13 \text{ (} 33.80 \text{ dB} \pm 0.02 \text{ dB})$$

$$T_{e,min} = 109.5 \text{ K} \pm 2.5 \text{ K}$$

$$F_{min} = 1.391 \text{ dB} \pm 0.027 \text{ dB}$$

$$t = 176.2 \text{ K} \pm 1.9 \text{ K}$$

$$\mathbf{G}_{opt} = (0.050 + 0.140j) \pm (0.013 + 0.010j)$$



- Dependence on uncertainty in hot noise temperature:

Table 3. Effect of Uncertainty in Noise Temperature of Hot Source.

S h,frac	$u_G$ (dB)	$u_{Tmin}$ (K)	$u_{Fmin}$ (dB)	$u_t$ (K)	$u_{ReG}$	$u_{ImG}$
0.005	0.024	2.5	0.027	1.9	0.013	0.011
0.010	0.048	4.6	0.050	2.6	0.013	0.011
0.020	0.096	8.9	0.098	4.2	0.013	0.011
0.0223 (0.1dB)	0.112	10.4	0.114	4.8	0.013	0.011
0.0351(0.15dB)	0.170	15.6	0.173	7.0	0.013	0.011



- Dependence on uncertainty in power measurement:

Table 1. Effect of Fractional Power Uncertainty.

$s_{p,\text{frac}}$	$u_G$ (dB)	$u_{T\min}$ (K)	$u_{F\min}$ (dB)	$u_t$ (K)	$u_{ReG}$	$u_{ImG}$
0.001	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.033	3.5	0.038	2.5	0.020	0.013
0.010	0.051	5.6	0.061	3.6	0.032	0.020



- Dependence on uncertainty in reflection coefficients of sources:

Table 2. Effect of Uncertainty in Reflection Coefficients.

$s_G$	$u_G$ (dB)	$u_{T\min}$ (K)	$u_{F\min}$ (dB)	$u_t$ (K)	$u_{ReG}$	$u_{ImG}$
0.002, 0.003	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.024	2.7	0.030	2.7	0.020	0.016
0.010	0.027	3.5	0.038	5.0	0.037	0.032



- Some representative cases
  - underlying uncertainties

Table 4. Underlying Uncertainties Used in Representative Cases

Case	$u_{Ta,frac}$	$u_{Th/c,frac}$	$u_{p,frac}$	$u_G$	$u_{con}$	$u_{S2I}$
Average	0.005	0.020	0.002	0.005	0.001	0.010
Good	0.005	0.005	0.001	0.002, 0.003	0.001	0.010
VG-h	0.001	0.005	0.001	0.002	0.001	0.010
VG-c	0.001	0.008	0.001	0.002	0.001	0.010



- Resulting uncertainties in noise parameters

Table 5. Noise Parameter Uncertainties for Representative Cases

Case	$u_G$ (dB)	$u_{Tmin}$ (K)	$u_{Fmin}$ (dB)	$u_t$ (K)	$u_{ReG}$	$u_{ImG}$
Average	0.101	9.0	.099	4.6	.020	.016
Good	0.024	2.5	.027	1.9	.013	.011
VG-h	0.024	2.4	.026	1.5	.008	.007
VG-c	0.019	1.5	.016	1.4	.008	.007



- Effect of omitting one termination

Omitted	$u_G$ (dB)	$u_{T_{min}}$ (K)	$u_{F_{min}}$ (dB)	$u_t$ (K)	$u_{ReG}$	$u_{ImG}$
None	0.024	2.5	0.027	1.9	0.013	0.011
2	0.024	2.5	0.028	1.9	0.013	0.010
3	0.024	2.5	0.027	1.9	0.014	0.011
4	0.024	2.9	0.032	1.9	0.015	0.011
5	0.024	2.5	0.027	2.2	0.014	0.011
6	0.024	2.5	0.027	2.0	0.014	0.012
7	0.024	2.6	0.028	1.9	0.013	0.011
8	0.024	2.5	0.027	2.2	0.013	0.014
9	0.024	2.5	0.027	1.9	0.013	0.010
10	0.024	3.0	0.032	3.0	0.028	0.034
11	0.025	2.7	0.029	1.9	0.019	0.011
12	0.024	2.6	0.026	2.0	0.016	0.013
13	0.024	2.5	0.027	2.0	0.014	0.010



## IV. DISCUSSION

- Note that this is for type-B uncertainties only; there will also be type-A uncertainties from the fit.
- General features:
  - noise-parameter uncerts not very sensitive to ambient temperature (**but device itself may be**)
  - $G$  &  $T_{min}$  uncerts controlled by  $T_h$  &  $P$  uncerts
  - ' $_{opt}$  uncerts controlled by ' $_{source}$  &  $P$  uncerts
  - $t$  uncertainty affected by everything.

- Shortcomings:
  - mismatch neglected in power measurements
  - lack of correlations
  - lack of other (non-Gaussian) distributions
- Future:
  - Remedy shortcomings **U(mostly)**
  - Include more general fitting routine **U(almost)**
  - More complete study (including reverse configuration), use to choose NIST strategy
  - Package and distribute, if sufficient interest exists